Shaping of photorefractive two-wave coupling by fast phase modulation

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Using the distinctive features of the photorefractive nonlinearity, we derive a general self-consistent set of equations to describe two-wave coupling in the presence of fast and arbitrary strong-phase modulation. By considering a number of important particular cases, we show that phase modulation is a powerful and useful tool for shaping the characteristics of two-wave coupling such as the value of the energy exchange, the diffraction efficiency of the recorded grating, and the structure of the grating fringes. Finally, we analyze the role of the phase modulation in the active stabilization of wave coupling by means of an electronically introduced phase feedback.

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I. INTRODUCTION

Photorefractive two-wave coupling is, as is well known, due to the dynamic processes of recording of a refractive index grating and diffraction from this grating [1,2]. The buildup of the grating is, in turn, owing to the charge separation by light and the linear electrooptic effect. The characteristic time of charge separation in continuous wave (CW) experiments ranges usually from $\sim 10^{-2}$ to $\sim 10^{2}$ s and the characteristic nonlinear length of phase changing may be less than $\sim 10^{-1}$ cm [1–3].

The main characteristics of the photorefractive wave coupling such as the value of the energy exchange, the diffraction efficiency of the recorded index grating, and the structure of the grating fringes are well understood, at least within the simplest models for light-induced charge transport. These characteristics depend essentially on the type of the photorefractive response, i.e., on the phase shift between the initiating light interference pattern and the corresponding index grating. The possibilities to affect two-wave coupling externally are rather restricted.

In this paper, we focus our attention on the possibility to shape the photorefractive two-wave (2W)-coupling by means of fast phase modulation of an input (signal) light beam. The essence of the influence may be explained as follows with the aid of Fig. 1.

The amplitude S of the modulated beam includes both the fast ($\tilde{S}$) and slow ($\bar{S}$) components whereas the amplitude of the reference beam R has initially no fast component. The light interference pattern (characterized by the product SR^*) experiences both fast oscillations and slow changes. Since the period of the phase modulation is supposed to be much shorter than the buildup time of the space-charge field, the fast oscillations of the interference fringes at first do not affect the grating formation. At the same time, the slowly varying grating allows mutual Bragg diffraction of the R and S beams. Thereby the amplitude R acquires a fast component ($\tilde{R}$) inside the crystal. Because the product $\tilde{S}\tilde{R}^*$ has now a slowly varying part, the fast components of the signal and reference beams start also to participate in the grating formation. One can expect that the phase modulation affects in this way the grating recording so that the grating fringes and the spatio-temporal behavior of the amplitudes R,S differ considerably from the conventional ones.

A few additional points related to the effect of phase modulation are worth noting:

The phase modulation technique is one of the simplest to introduce into the experimental setup [4]. The shape and the strength of the phase oscillations may be varied over wide limits.

The effect of fast modulation on 2W-coupling admits a complete analytic investigation on the basis of the most general properties of the photorefractive nonlinearity.

Phase modulation has an important implication for the active stabilization of two-wave coupling by means of an electronically introduced phase feedback between the input and the output [5]. Experiment and numerical modeling have shown [5–8] that a proper feedback leads often to a periodic

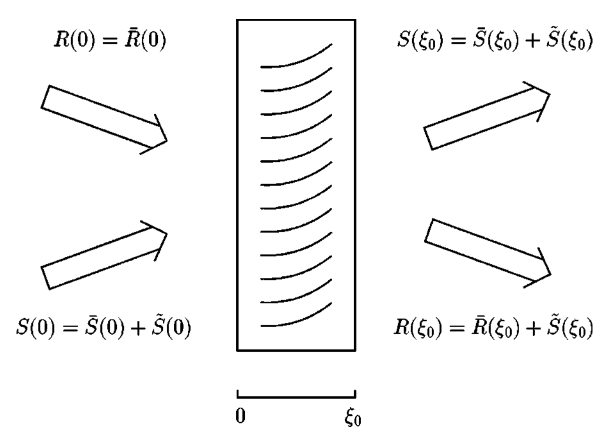

FIG. 1. Scheme of a photorefractive 2W-coupling experiment. The parallel bended lines show the grating fringes.

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state with practically transparent (or totally diffracting) index grating. This exciting property is definitely caused by the effect of phase modulation of the input signal beam.

Note that the effect of fast phase modulation on photorefractive wave mixing was already considered in a number of papers [9–14] having in mind possible applications. The authors of Refs. [9–11] analyze the effect of fast phase modulation on four-wave mixing within the undepleted pump approximation. On the basis of this analysis they demonstrated a photorefractive lock-in detector, a frequency converter, and a phase sensitive detector. An exact steady-state solution for the simplest four-wave configuration with phase-modulated input beams was studied in Ref. [12]. The effect of fast phase modulation on two-wave coupling was considered earlier in Ref. [13] within a particular mathematical model.

The method introduced in our paper is new to the best of our knowledge; it is essentially more general and simpler in form in comparison with the methods use in Refs. [12] and [13]. The applications of our method are different from those considered in Refs. [9–11].

II. GENERAL RELATIONS

A. Equations for two-wave coupling

The initial set of equations for the photorefractive 2W-coupling can be written in the following dimensionless form, see Refs. [2] and [8]:

$$\partial_\xi R = iES \tag{1}$$

$$\partial_\xi S = iE^* R \tag{2}$$

$$(e^{i\delta}\partial_\tau + 1)E = e^{i\theta} RS^*. \tag{3}$$

Here ξ and τ are the dimensionless coordinate and time, R and S are the dimensionless complex amplitudes of the reference and signal beams (see also Fig. 1), E is the dimensionless grating amplitude, θ and δ are the characteristic phases, and the asterisk means, as usual, complex conjugation. It is supposed that $0 \leqslant \xi \leqslant \xi_0$, where ξ_0 is the dimensionless crystal thickness; in experiment often $\xi_0 \gg 1$.

Actually, the set (1)–(3) is valid for many particular microscopic models of the photorefractive nonlinearity. The specification of the introduced dimensionless parameters for the simplest models may easily be found using Refs. [1–3]. The importance of the parameters θ and δ is different. The phase θ characterizes the type of the photorefractive response and ranges from 0 to 2π. For the diffusion charge transport $\theta = \pm\pi/2$; in the case of dominating drift in an external field or of photovoltaic transport θ is often near 0 or π. The cases $\theta = 0$ and $\theta = \pm\pi/2$ are often referred to as the cases of the local and nonlocal photorefractive response. A nonzero value of the phase δ means merely that the relaxation rate of the space-charge field is a complex quantity; often δ is near zero. In the general case $|\delta| < \pi/2$.

Equations (1) and (2) describe mutual Bragg diffraction of the R and S beams from the grating. The total light intensity remains obviously constant during propagation; without loss of generality we suppose that the dimensionless total intensity $|R|^2 + |S|^2 = 1$. Equation (3) describes evolution of the grating subject to the interference light pattern. The grating fringes are generally tilted and bent and they are changing in time. In spite of its simplicity the nonlinear set (1)–(3) cannot generally be solved analytically; it allows a great deal of nonlinear regimes for the light amplitudes R and S.

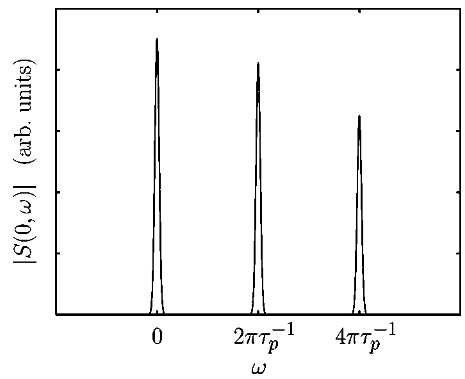

FIG. 2. Scheme of the Fourier spectrum of the input signal.

Let the R and S beams be incident onto the plane $\xi = 0$ as shown in Fig. 1. Then, we can formulate the boundary conditions for Eqs. (1)–(3) in the form

$$R(0,\tau) = R_0; \quad S(0,\tau) = S_0 \exp(i\varphi_p), \tag{4}$$

where the phase $\varphi_p = \varphi_p(\tau)$ is a periodic function with period $\tau_p \ll 1$ and a zero average value, $\langle \varphi_p \rangle = 0$. This function describes the fast phase modulation in question. The input amplitudes R_0 and S_0 are generally slowly varying functions of τ.

Figure 2 shows schematically the Fourier spectrum of the input signal $S(0,\omega)$. The peaks placed at approximately $2\pi\tau_d^{-1}$, $4\pi\tau_d^{-1}, \ldots$ are indeed due to the fast phase modulation in general. They are not much smaller than the zero peak. The width of the peaks, which is supposed to be of the order of one, is due to slow changes of S_0. The output amplitudes are expected to have a similar structure of the Fourier spectrum.

B. Fundamental solutions

The first two equations of the set (1)–(3) do not contain any time derivatives and are linear in R,S. This property enables us to formulate some fundamental features of the diffraction process.

Let us consider the grating amplitude E in Eqs. (1) and (2) as a certain single-valued function of ξ at an arbitrary moment τ. We can claim then that these equations possess the fundamental solution $\mathcal{R}_r, \mathcal{S}_r$ meeting the boundary conditions $\mathcal{R}_r(0) = 1$, $\mathcal{S}_r(0) = 0$. This solution corresponds physically to a test of the spatial grating (recorded up to the moment τ) by a single beam of unit amplitude incident in the R direction, see Fig. 3(a); $\mathcal{R}_r(\xi)$ and $\mathcal{S}_r(\xi)$ are the transmitted and diffracted parts of this test beam, respectively. We can define analogously another independent fundamental solution of the set (1), (2), $\mathcal{R}_s, \mathcal{S}_s$, which meets the boundary conditions $\mathcal{R}_s(0) = 0$, $\mathcal{S}_s(0) = 1$ and corresponds to a test of the same grating by a single beam of unit amplitude incident in the S direction, see Fig. 3(b). Using Eqs. (1) and (2), one can come easily to the important relations

$$\mathcal{S}_s = \mathcal{R}_r^*; \quad \mathcal{R}_s = -\mathcal{S}_r^*. \tag{5}$$

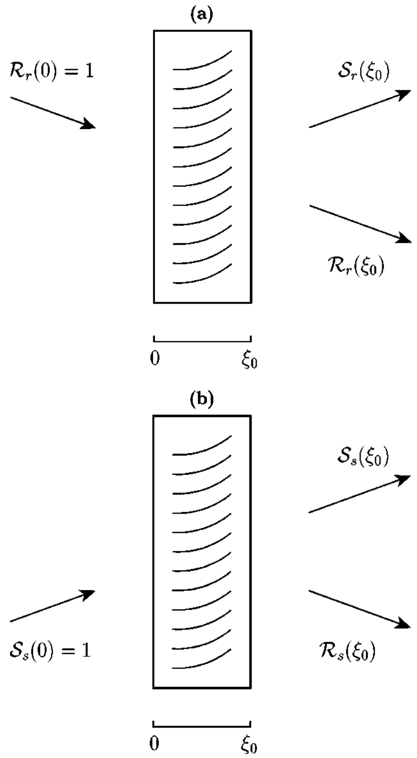

FIG. 3. Geometric schemes for the introduction of the fundamental solutions.

They express the symmetry of Eqs. (1) and (2) under interchange of the beams R and S. The quantity $\eta=|\mathcal{R}_s(\xi_0)|^2=|\mathcal{S}_r(\xi_0)|^2$ is nothing else than the diffraction efficiency of the grating at the moment τ.

The solution of Eqs. (1) and (2) with boundary conditions (4) is expressed by the fundamental solutions as follows:

$$R=R_0\mathcal{R}_r+S_0 e^{i\varphi_p}\mathcal{R}_s$$
$$S=R_0\mathcal{S}_r+S_0 e^{i\varphi_p}\mathcal{S}_s. \tag{6}$$

Equations (6) show explicitly that each of the amplitudes consists of transmitted and diffracted components. The found representation allows, as we shall see in the next section, to perform the averaging of Eqs. (1)–(3) over the fast phase oscillations.

III. TIME AVERAGING

As follows from the structure of Eq. (3), the response of the grating amplitude E to the temporal oscillation of the product RS^* with frequencies much higher than 1 (the high-frequency peaks in Fig. 2) is strongly suppressed. In other words, in the leading approximation in τ_p the grating amplitude E is a slowly varying function of τ despite of possibly strong fast oscillations of R and S. Correspondingly, the fundamental solutions $\mathcal{R}_{r,s}$ and $\mathcal{S}_{r,s}$ are also slowly varying functions of τ.

To use this fact for the averaging, we present the light amplitudes R and S in the form

$$R=\overline{R}+\widetilde{R}$$
$$S=\overline{S}+\widetilde{S}, \tag{7}$$

where $\overline{R},\overline{S}$ and $\widetilde{R},\widetilde{S}$ are the slow and fast components, respectively, so that $\langle\widetilde{R}\rangle=\langle\widetilde{S}\rangle=0$. Then we have from Eqs. (1)–(3) for $\overline{R}$ and $\overline{S}$,

$$\partial_\xi\overline{R}=iE\overline{S}$$
$$\partial_\xi\overline{S}=iE^*\overline{R}. \tag{8}$$

These equations are not different in structure from Eqs. (1) and (2). As follows from Eq. (8), the sum $I_0=|\overline{R}|^2+|\overline{S}|^2\leqslant 1$ does not depend on the propagation coordinate ξ.

For the grating amplitude we have after averaging of Eq. (3),

$$(e^{i\delta}\partial_\tau+1)E=e^{i\theta}(\overline{R}\overline{S}^*+\langle\widetilde{R}\widetilde{S}^*\rangle). \tag{9}$$

The specific features of the case under study come from the average $\langle\widetilde{R}\widetilde{S}^*\rangle$ on the right-hand side of this equation.

Our next goal is to express $\langle\widetilde{R}\widetilde{S}^*\rangle$ by $\overline{R}$ and $\overline{S}$. Since R_0 and S_0 have no fast components, we get from Eq. (6),

$$\overline{R}=R_0\mathcal{R}_r+\varepsilon S_0\mathcal{R}_s \tag{10}$$

$$\overline{S}=R_0\mathcal{S}_r+\varepsilon S_0\mathcal{S}_s \tag{11}$$

$$\widetilde{R}=S_0\mathcal{R}_s[\exp(i\varphi_p)-\varepsilon] \tag{12}$$

$$\widetilde{S}=S_0\mathcal{S}_s[\exp(i\varphi_p)-\varepsilon], \tag{13}$$

where $\varepsilon=\langle\exp(i\varphi_p)\rangle$ is a complex parameter characterizing the effect of phase modulation. Obviously, we have $|\varepsilon|\leqslant 1$ and the limiting case $|\varepsilon|=1$ corresponds to the absence of the fast modulation. Using Eqs. (5), (10), and (11), we express $\mathcal{S}_s^*$ and $\mathcal{R}_s$ algebraically by $\overline{R}$ and $\overline{S}^*$,

$$\mathcal{S}_s^*=(R_0^*\overline{R}+\varepsilon S_0\overline{S}^*)I_0^{-1} \tag{14}$$

$$\mathcal{R}_s=(\varepsilon^* S_0^*\overline{R}-R_0\overline{S}^*)I_0^{-1} \tag{15}$$

with $I_0=|R_0|^2+|\varepsilon S_0|^2\equiv|\overline{R}|^2+|\overline{S}|^2$. Substituting finally these expressions into Eqs. (12) and (13), we obtain easily,

$$\langle\widetilde{R}\widetilde{S}^*\rangle=|S_0|^2 I_0^{-2}(1-|\varepsilon|^2)(R_0^*\overline{R}+\varepsilon S_0\overline{S}^*)$$
$$\times(\varepsilon^* S_0^*\overline{R}-R_0\overline{S}^*). \tag{16}$$

This is the relation sought for. For $|\varepsilon|^2=1$, which means no fast modulation, we have from Eq. (16) the obvious result, $\langle\widetilde{R}\widetilde{S}^*\rangle=0$.

By substituting Eq. (16) into Eq. (9), we arrive at the following explicit equation for E, completing the set (8):

$$(e^{i\delta}\partial_\tau+1)E=e^{i\theta}[\alpha\overline{R}\overline{S}^*-\beta(\overline{S}^*)^2+\beta^*\overline{R}^2], \tag{17}$$

where

$$\alpha = 1 + |S_0|^2(1-|\varepsilon|^2)(|\varepsilon S_0|^2 - |R_0|^2)I_0^{-2}$$
$$\beta = |S_0|^2(1-|\varepsilon|^2)\varepsilon S_0 R_0 I_0^{-2}. \tag{18}$$

The parameters α and β are fully defined by ε and the slow input amplitudes S_0 and R_0. The boundary values of $\overline{S}(\xi)$ and $\overline{R}(\xi)$ are obviously given by $\overline{S}(0) = \varepsilon S_0$ and $\overline{R}(0) = R_0$.

The terms proportional to $(\overline{S}^*)^2$ and $\overline{R}^2$ make the structure of Eq. (17) essentially different from that of the conventional equation for the grating amplitude [1–3]. We can expect therefore that 2W-coupling in photorefractive crystals can be shaped effectively by the fast phase modulation.

Note that the fast components $\tilde{R}, \tilde{S}$ as well as the total intensities of the R and S beams may be expressed through $\overline{R}$, $\overline{S}$ using Eqs. (12)–(15). The diffraction efficiency, $\eta = |\mathcal{R}_s(\xi_0)|^2$, is expressed by $\overline{R}$, $\overline{S}$ using Eq. (15).

IV. THE UNITARY TRANSFORMATION

Equation (8) for $\overline{R}$, $\overline{S}$ possess a fundamental symmetry property that can be verified by direct calculation. They are invariant under the unitary transformation $\overline{R},\overline{S} \to u, v$ given by

$$\begin{pmatrix} \overline{R} \\ \overline{S}^* \end{pmatrix} = \begin{pmatrix} P & -Q^* \\ Q & P^* \end{pmatrix} \begin{pmatrix} u \\ v^* \end{pmatrix}, \tag{19}$$

where P, Q^* are complex parameters meeting the condition $|P|^2 + |Q|^2 = 1$. In other words, after this transformation we have

$$\partial_\xi u = iEv$$
$$\partial_\xi v = iE^* u. \tag{20}$$

Physically, this means that there is a variety of wave pairs of the same total intensity coupled via the same grating.

Equation (17) for the grating amplitude changes its structure under the unitary transformation. Moreover, by a proper choice of P and Q this equation may be considerably simplified. This gives us a constructive method for solving the set (8), (17).

After the unitary transformation, the right-hand side of Eq. (17) contains terms proportional to uv^*, u^2, and v^{*2}. By imposing the additional condition,

$$\frac{Q}{P} = \frac{\alpha \pm \sqrt{\alpha^2 + 4|\beta|^2}}{2\beta}, \tag{21}$$

we eliminate simultaneously the terms u^2 and v^{*2} so that the equation for E attains the form

$$(e^{i\delta}\partial_\tau + 1)E = \mp \sqrt{\alpha^2 + 4|\beta|^2} e^{i\theta} u v^*. \tag{22}$$

The closed set of Eqs. (20) and (22) for u, v, and E is not different in structure from the conventional one for 2W-coupling. The effect of the fast modulation comes now from the square root in Eq. (22) and from the algebraic relations between u, v and $\overline{R}, \overline{S}$. As can be shown, both signs in Eqs. (21) and (22), the upper and the lower, lead to the same physical results. For definiteness sake, we choose from now on the upper sign in these equations.

The explicit expressions for P and Q may then be chosen in the form

$$P = \frac{1}{\sqrt{2}} \left(1 - \frac{\alpha}{\sqrt{\alpha^2 + 4|\beta|^2}} \right)^{1/2} \frac{R_0}{|R_0|}$$
$$Q^* = \frac{1}{\sqrt{2}} \left(1 + \frac{\alpha}{\sqrt{\alpha^2 + 4|\beta|^2}} \right)^{1/2} \frac{\varepsilon S_0}{|\varepsilon S_0|}. \tag{23}$$

The boundary values of new variables at $\xi = 0$, u_0 and v_0, are given by the inverse unitary transformation,

$$\begin{pmatrix} u_0 \\ v_0^* \end{pmatrix} = \begin{pmatrix} P^* & Q^* \\ -Q & P \end{pmatrix} \begin{pmatrix} \overline{R}_0 \\ \overline{S}_0^* \end{pmatrix}. \tag{24}$$

The relations derived in this section are sufficient to describe fully 2W-coupling under fast phase modulation.

V. STEADY-STATE SOLUTIONS

In the general case, a frequency shift Ω exists in steady state between the interacting waves. This shift is supposed to be sufficiently small, $|\Omega| \lesssim 1$. The grating fringes are moving in steady state with constant velocity.

To obtain the stationary solution of Eqs. (20) and (22) for u, v^* we assume that R_0 and $\overline{R}$ are constant in time while S_0 and $\overline{S}$ are proportional to $\exp(i\Omega\tau)$. Then we see from Eqs. (19), (20), (22), and (23) that $P, u = \text{const}$ and $Q, v^*, E \propto \exp(-i\Omega\tau)$. Correspondingly, we have $E = -i\gamma u v^*/I_0$ and the following ordinary differential equations for u and v^*:

$$I_0 u_\xi = \gamma u |v|^2$$
$$I_0 v_\xi^* = -\gamma v^* |u|^2, \tag{25}$$

where

$$\gamma = g\Psi \tag{26}$$

is a product of two different factors. The positive factor g depends on the input parameter $W_0 = |R_0|^2 - |S_0|^2$ and $|\varepsilon|^2$,

$$g = \sqrt{W_0^2 + |\varepsilon|^2(1 - W_0^2)}. \tag{27}$$

This factor is symmetric under the interchange of the input intensities $|R_0|^2$ and $|S_0|^2$. It is obviously an increasing function of $|\varepsilon|^2$. For $|\varepsilon|^2 \simeq 1$ or for $W_0 \simeq \pm 1$ (strongly different input intensities) we have $g \simeq 1$. Note that we prefer to use the input parameter W_0, which ranges from -1 to 1, instead of the input contrast of the interference pattern, $\sqrt{1 - W_0^2}$. Sometimes, we shall also use the input beam ratio $r_0 = |R_0|^2/|S_0|^2 = (1 + W_0)/(1 - W_0)$ instead of W_0. This parameter ranges from zero to infinity.

The complex parameter Ψ depends on the frequency detuning Ω and the characteristic phases θ and δ,

$$\Psi = \frac{i \exp(i\theta)}{1 - i\Omega \exp(i\delta)}. \tag{28}$$

The solution of Eqs. (25) with the boundary values $v_0^* = v^*(0)$, $u_0 = u(0)$ has the form

$$u = u_0 \left[\frac{I_0}{|u_0|^2 + |v_0|^2 \exp(2\gamma' \xi)} \right]^{\gamma/2\gamma'} \quad (29)$$

$$v^* = v_0^* \left[\frac{I_0}{|v_0|^2 + |u_0|^2 \exp(-2\gamma' \xi)} \right]^{\gamma/2\gamma'},$$

where, $\gamma' = \Re\gamma$. As follows from Eq. (29) [or from Eq. (19)], $|u|^2 + |v|^2 = |u_0|^2 + |v_0|^2 \equiv I_0$.

For the amplitudes $\overline{R}$, $\overline{S^*}$ we have after some calculations with the use of Eqs. (19), (23), and (24):

$$\overline{R} = R_0 \left[\cosh\left(\frac{\gamma\xi}{2}\right) + \frac{(W_0 + |\varepsilon|^2 - W_0|\varepsilon|^2)}{g} \sinh\left(\frac{\gamma\xi}{2}\right) \right] \times \left[\cosh(\gamma'\xi) + \frac{W_0}{g} \sinh(\gamma'\xi) \right]^{-\gamma/2\gamma'}$$

$$\overline{S^*} = \varepsilon^* S_0^* \left[\cosh\left(\frac{\gamma\xi}{2}\right) - \frac{1}{g} \sinh\left(\frac{\gamma\xi}{2}\right) \right] \times \left[\cosh(\gamma'\xi) + \frac{W_0}{g} \sinh(\gamma'\xi) \right]^{-\gamma/2\gamma'}. \quad (30)$$

At $\varepsilon = 1$ the obtained relations simplify to the conventional ones for 2W-coupling [1–3]. At $\varepsilon = 0$, which means no slow input component of the S beam, we have $\overline{S} = 0$, $\overline{R} = R_0$, and $E = 0$. This result is expected because the light interference pattern has here only a fast component. It is not difficult to make sure also that $|\overline{R}|^2 + |\overline{S}|^2 = I_0$.

Using Eqs. (15) and (30), we can calculate now the diffraction efficiency of the recorded grating, $\eta = |\mathcal{R}_s(\xi_0)|^2$,

$$\eta = \frac{|\varepsilon|^2(1 - W_0^2)}{g} \frac{|\sinh(\gamma\xi_0/2)|^2}{g\cosh(\gamma'\xi_0) + W_0 \sinh(\gamma'\xi_0)}. \quad (31)$$

At $\varepsilon = 1$ this transforms again into the known result of Ref. [15]; at $\varepsilon = 0$ we have, as expected, $\eta = 0$.

The averaged output intensities $\langle |\tilde{R}(\xi_0)|^2 \rangle$ and $\langle |\tilde{S}(\xi_0)|^2 \rangle$, related to the quickly oscillating components of the beams, are expressed by η,

$$\langle |\tilde{R}(\xi_0)|^2 \rangle = |S_0|^2 \eta (1 - |\varepsilon|^2)$$

$$\langle |\tilde{S}(\xi_0)|^2 \rangle = |S_0|^2 (1 - \eta)(1 - |\varepsilon|^2). \quad (32)$$

The relations (30)–(32) include a great deal of information about the effect of fast phase modulation on the characteristics of 2W-coupling. In the following two sections, we apply these general relations to the most important particular cases.

VI. CHARACTERISTICS OF THE ENERGY EXCHANGE

The simplest results take place for the total time-averaged intensities, $\langle |R|^2 \rangle = |\overline{R}|^2 + \langle |\tilde{R}|^2 \rangle$ and $\langle |S|^2 \rangle = |\overline{S}|^2 + \langle |\tilde{S}|^2 \rangle$. Since $\langle |R|^2 \rangle + \langle |S|^2 \rangle = 1$, the above intensities may be fully characterized by a single combination of $\langle |R|^2 \rangle$ and $\langle |S|^2 \rangle$. It is useful to choose for this combination the beam ratio $r = \langle |R|^2 \rangle / \langle |S|^2 \rangle$. From Eqs. (30), (31), and (32) one can obtain,

$$\frac{r}{r_0} = \frac{1 + a_+ \tanh(\gamma'\xi)}{1 + a_- \tanh(\gamma'\xi)}, \quad (33)$$

where $r_0 = |R_0|^2 / |S_0|^2$ is again the input beam ratio and the real parameters $a_\pm$, such that $a_- < a_+$ and $|a_\pm| < 1$, are given by

$$a_+ = \frac{r_0 - 1 + 2|\varepsilon|^2}{\sqrt{(r_0 - 1)^2 + 4r_0|\varepsilon|^2}}$$

$$a_- = \frac{r_0 - 1 - 2r_0|\varepsilon|^2}{\sqrt{(r_0 - 1)^2 + 4r_0|\varepsilon|^2}}. \quad (34)$$

The spatial dependence of r is defined only by the real part of the characteristic exponent γ. Therefore, the energy exchange remains unidirectional irrespectively of the value of the parameter $|\varepsilon|^2$, characterizing the phase modulation. The sign of γ', as seen from Eqs. (26) and (27), does not depend on ε. If $\gamma' > 0$, energy is transferred to the R beam. In the absence of the fast modulation ($|\varepsilon|^2 = 1$) Eq. (33) gives the exponential spatial amplification, $r = r_0 \exp(2\gamma'\xi_0)$ of conventional for 2W-coupling. In the case of equal input intensities, $W_0 = 0$ ($r_0 = 1$), we have $a_\pm = \pm 1$, $\gamma' = |\varepsilon|\Psi'$, so that Eq. (33) is reduced to $r = \exp(|\varepsilon|\Psi'\xi_0)$.

The effect of phase modulation on the total intensities $\langle |R|^2 \rangle$ and $\langle |S|^2 \rangle$ is reduced to weakening of the energy transfer. Depending on the purpose of experiment, this effect may be regarded as positive or negative. Below we analyze the dependence $r(|\varepsilon|^2, \gamma'\xi_0, r_0)$ in more detail.

Let us consider the spatial amplification of a weak input R beam ($r_0 \ll 1$) in a sufficiently thick ($2\gamma'\xi_0 \gg 1$) nonlinear medium. In this case, we have from Eqs. (33) and (34):

$$\frac{r(\xi_0)}{r_0} \simeq \frac{|\varepsilon|^2 \exp(2\gamma'\xi_0) + 1}{r_0^2 |\varepsilon|^2 (1 - |\varepsilon|^2) \exp(2\gamma'\xi_0) + 1}, \quad (35)$$

where the dependence of γ' on $|\varepsilon|^2$ and r_0 is negligible, see Eq. (27). The main effect of the phase modulation comes from the denominator of Eq. (35) and becomes important for $r_0^2 |\varepsilon|^2 (1 - |\varepsilon|^2) \gtrsim \exp(-2\gamma'\xi_0)$. In the vicinity of the points $|\varepsilon|^2 = 0$ and 1 one can expect peculiarities of the spatial amplification.

Figure 4 shows the dependence $r(\xi_0)/r_0$ plotted on the basis of Eq. (33) for a small input beam ratio r_0 and several representative values of $|\varepsilon|^2$. The smaller $|\varepsilon|^2$, the lower is the corresponding curve. For a moderate crystal thickness, $\gamma'\xi_0 \lesssim 5$, decrease of $|\varepsilon|^2$ from 1 to $\simeq 0.5$ does not produce any sharp decrease of r. For $\gamma'\xi_0 \gtrsim 8$ the output beam ratio $r(\xi_0)$ becomes, however, highly sensitive to a decrease of $|\varepsilon|^2$ in the vicinity of unity. In other words, even a very weak phase modulation can strongly suppress the energy transfer. As seen from Fig. 4, for $\gamma'\xi_0 \gtrsim 6$ the dependence $r(|\varepsilon|^2)$ is also very sharp in the vicinity of zero. To make the energy

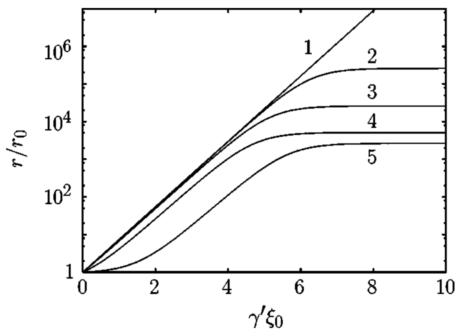

FIG. 4. Dependence of the ratio r/r_0 on the crystal thickness ξ_0 for the input beam ratio $r=0.02$. The curves 1, 2, 3, 4, and 5 correspond to $|\varepsilon|^2=1$, 0.99, 0.1, 0.5, and 0.05, respectively.

transfer very week, we have to adjust $|\varepsilon|^2$ to zero with a very high accuracy. The smaller the input ratio, the larger the critical crystal thickness and the higher is the saturated value of $r(\xi_0)/r_0$.

Figure 5 shows the dependence $r(r_0)$ for $\gamma'\xi_0=6$ and several values of $|\varepsilon|^2$. For $|\varepsilon|^2=1$ this dependence is indeed strictly linear. Introduction of the phase modulation changes the structure of the curves. For $|\varepsilon|^2<1$ the function $r(r_0)$ has a clear maximum. The position of this maximum shifts gradually to zero with decreasing $|\varepsilon|^2$ and its value is strongly decreasing.

In such a way, the phase modulation allows to shape considerably the characteristics of the spatial amplification. A sufficiently thick crystal acts indeed as a strongly nonlinear filter for amplifying signals.

The energy exchange between the slow intensity components $|\overline{R}|^2$ and $|\overline{S}|^2$, which can be separated from $|R|^2$ and $|S|^2$ by temporal filtering, is possible even for $\gamma'=0$ when the total intensities $\langle|R|^2\rangle$ and $\langle|S|^2\rangle$ remain constant. This case, relevant to dominating photovoltaic or drift charge transport in ferroelectrics [2,3], deserves a more detailed consideration.

By setting $\gamma'=0$ in Eq. (30) one can represent the spatial dependences of $|\overline{R}(\xi_0)|^2$ and $|\overline{S}(\xi_0)|^2$ in the form

$$|\overline{R}(\xi_0)|^2=|\overline{R}(0)|^2-\Delta\sin^2(\gamma''\xi_0/2)$$
$$|\overline{S}(\xi_0)|^2=|\overline{S}(0)|^2+\Delta\sin^2(\gamma''\xi_0/2), \tag{36}$$

where $\overline{R}(0)=R_0$, $\overline{S}(0)=\varepsilon S_0$, and

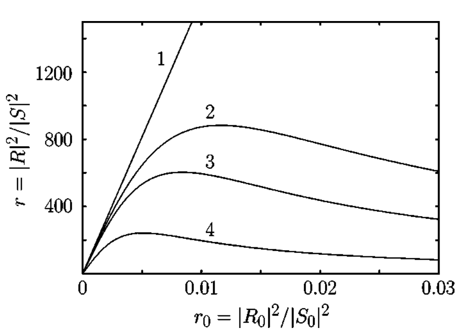

FIG. 5. The output beam ratio r versus r_0 for $\gamma'\xi_0=6$; the curves 1, 2, 3, and 4 are plotted for $|\varepsilon|^2=1$, 0.95, 0.9, and 0.6, respectively.

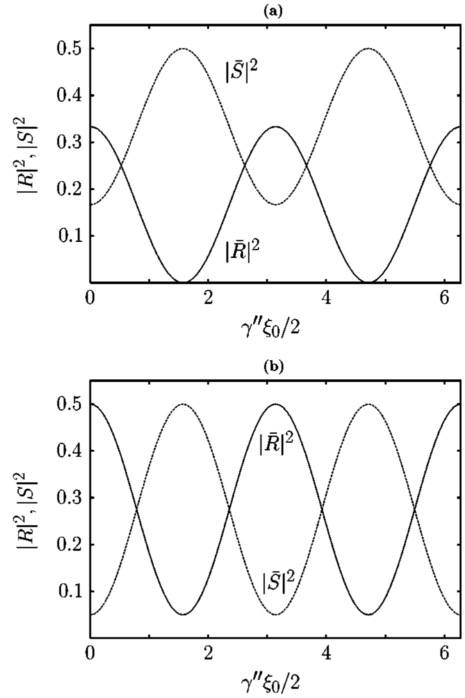

FIG. 6. The energy exchange between the slow intensity components for $\gamma'=0$. The solid and dashed lines correspond to $|\overline{R}|^2$ and $|\overline{S}|^2$, respectively. In the case (a) $|\varepsilon|^2=0.25$, $r_0=0.5$ and in the case (b) $|\varepsilon|^2=0.1$, $r_0=1$.

$$\Delta=\frac{4|R_0|^2|\varepsilon|^2(1-|\varepsilon|^2)}{(r_0-1)^2+4r_0|\varepsilon|^2}. \tag{37}$$

Since $\Delta>0$, the intensities $|\overline{R}|^2$, $|\overline{S}|^2$ oscillate with $\gamma''\xi_0$ between the input values and $|\overline{R}(0)|^2-\Delta$, $|\overline{S}(0)|^2+\Delta$, respectively, see Fig. 6. In average, the value of $|\overline{R}(\xi_0)|^2$ is less than $|\overline{R}(0)|^2$ whereas $|\overline{S}(\xi_0)|^2$ is larger than $|\overline{S}(0)|^2$.

The span Δ may approach (or be equal to) $|\overline{R}(0)|^2$ or $|\overline{S}(0)|^2$, which means a strong energy exchange. The equality $\Delta=|\overline{R}(0)|^2\equiv|R_0|^2$ takes place for $|\varepsilon|^2\leqslant 1/2$, $r_0=1-2|\varepsilon|^2<1$, see Fig. 6(a). The case $\Delta\simeq|\overline{S_0}|^2\equiv|\varepsilon|^2|S_0|^2$ corresponds to $r_0\simeq 1$, $|\varepsilon|^2\ll 1$; here $|\overline{S}(0)|^2\ll|S_0|^2\simeq|R_0|^2$, see Fig. 6(b).

In the general case, when both γ' and γ'' are nonzero, the energy exchange between $|\overline{R}|^2$ and $|\overline{S}|^2$ becomes more complicated. The spatial oscillations are superimposed here on the unidirectional energy transfer.

VII. FULLY DIFFRACTIVE AND TRANSPARENT GRATINGS

The diffraction efficiency of the recorded grating, η, is also an important characteristic of 2W-coupling. Below we show that η can be made equal to 1 or 0 with the use of the phase modulation technique for a sufficiently large crystal thickness ξ_0. In other words, the grating can be made fully diffractive or fully transparent.

To prove that it is possible to reach the ultimate value $\eta=1$, we use in Eq. (31) the identity $|\sinh(\gamma\xi_0/2)|^2=[\cosh(\gamma'\xi_0)-\cos(\gamma''\xi_0)]/2$. The presence of two variable parameters, $|\varepsilon|^2$ and Ω, in the expressions for γ [see Eqs. (27) and (28)] allows to consider $\gamma'\xi_0$ and $\gamma''\xi_0$ as two independent variables to maximize η. One can check directly that $\eta=1$ for

$$\gamma''\xi_0=\pi j$$
$$\gamma'\xi_0=L, \tag{38}$$

where j is an odd number (positive or negative) and $L=\ln[(g-W_0)/(g+W_0)]$. Using Eq. (26), these conditions can be represented in the form of relations expressing ξ_0 and Ω through $|\varepsilon|^2$ and the input beam ratio r_0 (or W_0). These relations include indeed the characteristic phases θ and δ, see Eqs. (3) and (30). The simplest relations correspond to the case $\delta=0$, most important for experiment. In this case we obtain instead of Eq. (38):

$$\xi_0=\frac{\pi^2 j^2+L^2}{g(\pi j\cos\theta-L\sin\theta)} \tag{39}$$

$$\Omega=-\frac{\pi j\tan\theta+L}{\pi j-L\tan\theta}. \tag{40}$$

Since $j=\pm 1,\pm 3,\ldots$, we have a sequence of branches for ξ_0 and Ω.

Let us consider in some detail what follows from Eqs. (39) and (40) for different values of the phase θ that characterizes the photorefractive response. The simplest case is $\theta=0$ (the local response). Here, the expression for ξ_0 is invariant under the reversal of the input beam ratio, i.e., it is even in $\log_{10} r_0$ (in W_0). Correspondingly, the function $|\varepsilon|^2(\log_{10} r_0)$ is also even. At the same time, the frequency detuning Ω is an odd function of $\log_{10} r_0$. The minimum possible value of the crystal thickness for $\theta=0$ is $\xi_0^{min}=\pi$; it matches Eq. (39) for $r_0=1$ and $\Omega=0$.

Figure 7(a) shows the dependences of $|\varepsilon|^2$ and Ω on $\log_{10} r_0$ for $\theta=0$, $j=1$, and different values of ξ_0. Since Eqs. (39) and (40) have no solutions for $j\neq 1$ within the range $\pi<\xi_0<3\pi$, the plotted curves cover adequately the case under study. We see that the permitted interval of r_0 expands quickly with increasing ξ_0; one can find that the extreme values of r_0 (that correspond to $|\varepsilon|^2=1$) are given by $|\log_{10} r_0|\simeq 0.43\sqrt{\pi(\xi_0-\xi_0^{min})}$. The minimum value of $|\varepsilon|^2(r_0)$ decreases rapidly with increasing ξ_0. For $\xi_0-\xi_0^{min}\ll\xi_0^{min}$ the dependence $\Omega(\log_{10} r_0)$ is nearly linear. With increasing ξ_0 it steepens near zero and saturates for sufficiently large values of $|\log_{10} r_0|$.

The case $\theta=\pi$ is not much different from the case $\theta=0$. We should choose here $j=1$ in Eqs. (39) and (40); therefore the dependences $|\varepsilon|^2(\log_{10} r_0)$ remain unchanged while $\Omega(\log_{10} r_0)$ has to be replaced by $-\Omega(\log_{10} r_0)$.

For $\theta\neq 0,\pi$ the dependences of $|\varepsilon|^2$ and Ω on $\log_{10} r_0$ lose their symmetry properties. These dependences change considerably when θ is increasing from 0 to $\pi/2$, see Figs. 7(b) and 7(c). One sees that the minimum crystal thickness increases and the actual range of $\log_{10} r_0$ shifts to the right. For $\theta=\pi/2$ (the diffusion like response) we have $\xi_0^{min}=2\pi$; this minimum thickness corresponds to $r_0=\exp(\pi)\simeq 23$ and $\Omega=-1$. The permitted values of r_0 are larger than 1 for $\theta=\pi/2$. Physically, this means that the total intensity of the weakest beam has to be increasing owing to 2W-coupling (the same holds true for $\theta=-\pi/2$). This property is clearly seen from Eq. (39) because ξ_0 becomes an odd function of W_0 (or $\log_{10} r_0$) for $\theta=\pi/2$. As seen from Fig. 7, the actual values of Ω decrease with increasing $\sin\theta$ and the curves that give $\Omega(\log_{10} r_0)$ for different values of ξ_0 tend to lose the intersection point. For $\theta=\pm\pi/2$ the branches with $j\neq 1$ come into the scene only for rather thick crystals, $\xi_0\geqslant 6\pi\simeq 19$.

Now we turn to the case $\eta=0$. Leaving aside the trivial case $\varepsilon=0$, one can see that this equality is equivalent to two requirements, $\gamma'=0$, $\gamma''\xi_0=2\pi n$, where n is an integer. By setting again $\delta=0$, we obtain, using Eq. (26), that $\Omega=-\tan\theta$ and

$$\xi_0=\frac{2\pi n}{g\cos\theta}. \tag{41}$$

These relations may be fulfilled for $\theta\neq\pm\pi/2$. The absolute minimum of the thickness, $\xi_0^{min}=2\pi$, corresponds to $|\varepsilon|^2=1$, $\Omega=0$, and $\cos\theta=\pm 1$. With $|\varepsilon|^2$ decreasing from one to zero ξ_0 is increasing by a factor of $1/|W_0|$. If $|W_0|<1/2(1/3<r_0<3)$, there are no forbidden gaps for ξ_0, i.e., for $\xi_0\geqslant 2\pi/|\cos\theta|$ one can find at least one proper value of $|\varepsilon|^2$ (and of Ω).

VIII. DISCUSSION

The generality of the proposed method for describing the effect of fast phase modulation can, in our opinion, be regarded as its most important distinctive feature. It is applicable, indeed, to all particular models of the photorefractive nonlinearity without any restrictions on the form and strength of the fast modulation.

By performing the time averaging, we did not use, in fact, the periodicity of the phase oscillations. Hence, the obtained results are applicable also to the effect of HF random phase fluctuations on 2W-coupling. The parameter ε is in this case nothing else than the statistical average of the corresponding phase exponent.

As a possibility for generalization of our method we can mention the introduction of a phase modulation into the second light beam. In this case not only averages like $\varepsilon=\langle\exp(i\varphi_p)\rangle$ but also the mutual correlation function of the corresponding exponential phase factors come onto the scene and 2W-coupling becomes possible even for $\varepsilon=0$.

Such a generalization is important, e.g., for the studies of the light-induced scattering inherent in most of the photorefractive crystals [1,2,16] and important for many practical applications. We hope that the phase modulation technique can be a useful tool for the experimental determination of the mechanisms and the correlation properties of this scattering in different crystals and optical configurations.

Owing to its generality, the proposed method can be applied to describe 4W-coupling, which is topical for analysis and optimization of various photorefractive schemes with the phase conjugation and optical oscillation [17,18]. The possibility to control the rate of spatial amplification and the diffractivity of the recorded grating looks attractive for the photorefractive devices based on 4W-mixing.

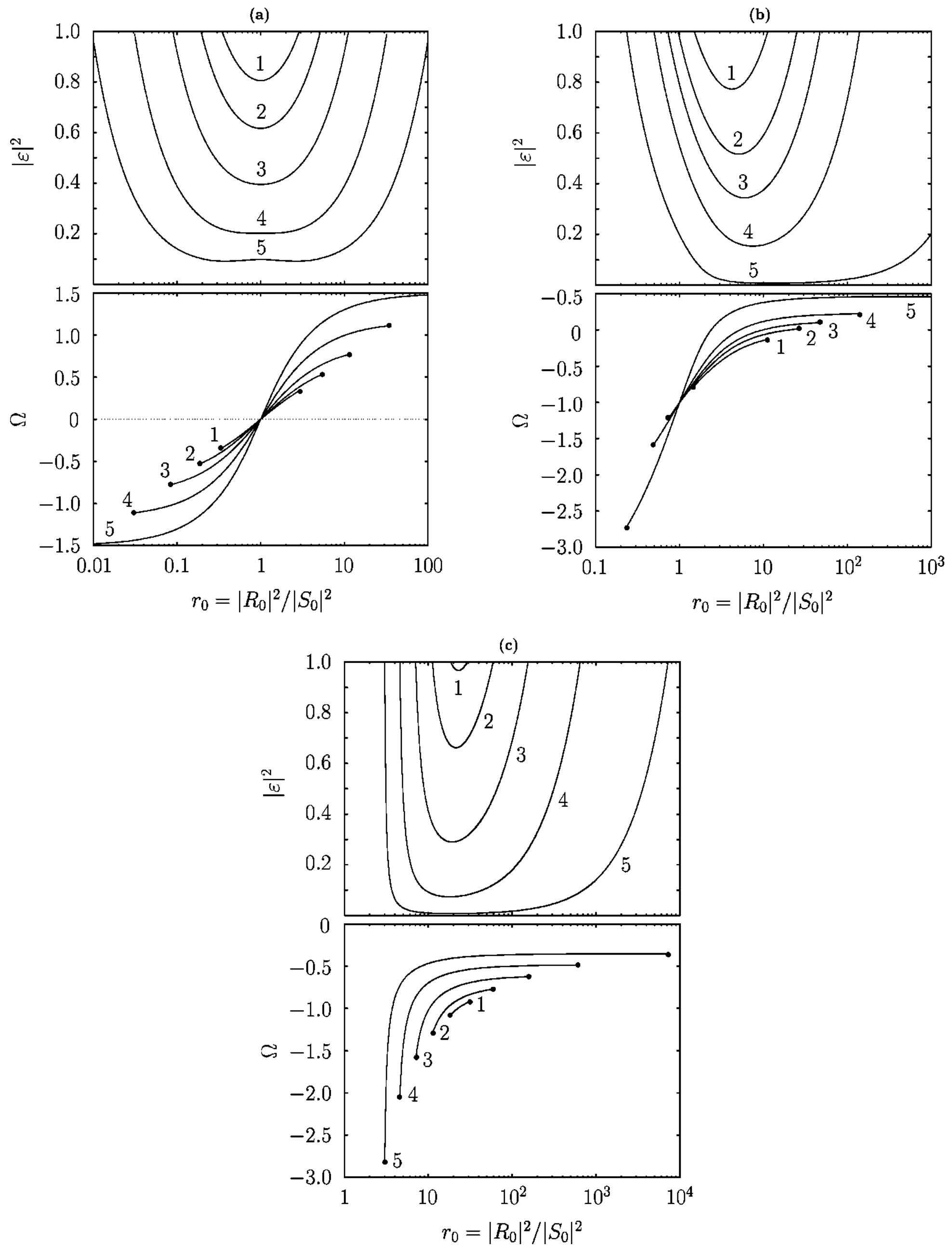

FIG. 7. Dependences $|\epsilon|^2(r_0)$ and $\Omega(r_0)$ for different values of θ and ξ_0. The cases (a), (b), and (c) correspond to $\theta=0$,45, and 90 degrees, respectively. The curves 1, 2, 3, 4, and 5 in the case (a) are plotted for $\xi_0=3.5$, 4, 5, 7, and 10. For the cases (b) and (c) the curves 1, 2, 3, 4, and 5 correspond to $\xi_0=4$, 4.5, 5, 6, and 10 and $\xi_0=6.3$, 6.5, 7, 8, and 10, respectively.

The developed theory has given a number of clear predictions for the effect of strong and fast phase modulation on the characteristics of 2W-coupling. We are expecting to see these predictions verified experimentally in the near future. Photorefractive ferroelectrics, such as $LiNbO_3$, $BaTiO_3$, and $KNbO_3$, with strong local and nonlocal nonlinear responses and relatively long response time seem to be most promising for this purpose.

It is important that the results of Sec. VII on maximization (minimization) of the diffraction efficiency have a close relation to the known experimental data on the feedback controlled 2W-coupling [5–7]. In this case, an electronic feedback loop adjusts the input phase of the signal beam to maintain a $\pi/2$ (or $-\pi/2$) phase shift between the diffracted and transmitted components of this beam. Both experiment [5–7] and numerical simulations [8] show that such a feedback brings a sufficiently thick crystal to the state with $\eta=1$ (or 0). Since the transmitted (or diffracted) component of the S beam becomes here zero, the above phase shift becomes undefined and the feedback fails. After that the system flees quickly the state with $\eta=1$ (or 0) and the feedback loop starts to operate again [8]. The periodic states described in Sec. VII should therefore be considered as the final result of introducing the $\pm\pi/2$ feedback. Its is remarkable that the proper values of ε and Ω are achieved automatically in this case. At the same time, the periodic states with $\eta=1$ and 0 cannot be described within the feedback model because the fast phase modulation arises owing to failure of the feedback at certain time moments. The obtained results form the firm basis for understanding of the known experimental results and of the consequences of introduction of the $\pm\pi/2$-feedback loop.

IX. CONCLUSIONS

Using the distinctive features of the photorefractive nonlinearity, we have proposed and developed a general method for describing 2W-coupling under a fast and arbitrary strong-phase modulation. The theory has given a number of predictions for shaping the characteristics of 2W-coupling and, in particular, for maximization and minimization of the diffraction efficiency. The connection with the prior experimental results on active stabilization is revealed. Possibilities for generalization and application of the obtained theoretical results are outlined.

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